

Challenge Problems from HKCEE Quick Drill text Book

Part 1: Trigonometry

1. Find the exact value of $(\cos 36^\circ)(\cos 108^\circ)$ without using a calculator
2. Prove that: $\tan(45^\circ + x) - \tan(45^\circ - x) = 2 \tan 2x$
3. Find the general solution of the equation: $\sin 9x \sin 8x - \sin 7x \sin 6x = 0$
4. Prove that: $\frac{\cos x - 5 \cos 5x + \cos 9x}{\sin x - 5 \sin 5x + \sin 9x} = \cot 5x$
5. Using the formula: $\cos 3x = \cos(x + 2x)$, prove that: $\cos 3x = 4 \cos^3 x - 3 \cos x$

a. What is the value of $\cos x$ if $\cos 3x = 1$

6. Show that $\sin 3x = 3 \sin x - 4 \sin^3 x$

a. Using the formula above, Prove that $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$

7. Given that: $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$, show that: $\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$

a. Using the formula above, solve $\tan^3 x - 3 \tan^2 x - 3 \tan x + 1 = 0$

8. Prove that: $\cot(x+y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$

a. Show that if $A+B+C = \pi$, where $0 < A, B, C < \pi$ then:
 $\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

b. Show that if $w + x + y + z = \pi$ and $0 < w, x, y, z < \pi$, then

$$\cot w \cot x \cot y + \cot w \cot x \cot z + \cot x \cot y \cot z + \cot w \cot y \cot z = \cot w + \cot x + \cot y + \cot z$$